Polarization Optimization of Compact Antenna Arrays for Direction of Arrival Estimation

Mariana Pralon, Matthias Hein and Reiner Thomä
Ilmenau University of Technology
Electronic Measurement Research Laboratory
Ilmenau, Germany
Email: {mariana.pralon, matthias.hein, reiner.thomae}@tu-ilmenau.de

Leandro Pralon, Bruno Pompeo and Gabriel Beltrao
Brazilian Army Technology Center
Rio de Janeiro, Brazil
Email: {pralon, pompeo, bgabriel}@ctex.eb.br

Giovanni Del Galdo and Markus Landmann
Fraunhofer Institute for Integrated Circuits
Ilmenau, Germany
Email: giovanni.delgaldo@tu-ilmenau.de, markus.landmann@iis.fraunhofer.de

***Abstract*—Analysis of polarization optimization of dual-polarized compact arrays for direction finding, which include Decoupling and Matching Networks (DMNs) comprised of distributed elements, is performed in this paper. The CRLB for two-dimensional Direction of Arrival (DoA) estimation is used as a figure of merit for the performance of five elements dual-polarized circular L-Quad arrays, assuming different inter-element spacings and the correspondent DMNs, designed with the aim of decoupling and matching the compact arrays, with inter-element spacing smaller than half of the free-space wavelength. Owing to the dual-polarimetric characteristic of the antenna array investigated, it is possible to optimize the DMN to decouple and match the array elements for the direction finding of impinging electromagnetic wavefronts.**

***Index Terms*—direction finding, source localization, dual-polarized antennas, CRLB, decoupling, matching, compact antenna arrays, decoupling networks.**

I. Introduction

In the late years, there has been a great amount of discussion and research in the academic community, as well as in industry, with respect to the overall size of devices used for communication based applications. The trend is to reduce the volume occupied the more the possible through miniaturization techniques. Since antennas are crucial to any communication system, in order to accomplish miniaturization it is of great importance to focus on the volume occupied by them, giving rise to two possible outcomes. First, it may be possible to miniaturize the antenna radiating element itself, and second, in case of antenna arrays, it may be required that the radiating elements are placed together more closely, in order to reduce the volume.

One of the most researched applications in the signal processing community for communication systems is the localization of emitters, and more specifically, the localization in terms of Direction of Arrival (DoA), also referred to as direction finding. Applications for direction finding include RADAR systems, satellite navigation, autonomous driving, among others. High-resolution direction finders employ antenna arrays comprised of several elements instead of a single radiating element to receive the impinging signals and subsequently feed the output to a DoA estimator. As a consequence, there has been an increased demand for highly capable DoA estimation antenna arrays.

With the aim of investigating the performance of arrays in terms of direction finding capabilities, parameters such as the Cramér-Rao Lower Bound (CRLB), reviewed in [1], are useful tools, as it tells the antenna designer how accurate the estimator can be when the array is used as part of the direction finding system [2].

This paper investigates compact antenna arrays, with inter-element spacing smaller than half of the free-space wavelength. It is well known that when designing antenna arrays, the optimal distance between elements, which ensures negligible mutual coupling while avoiding ambiguity, is half of the wavelength in the free-space. If we reduce this spacing, adverse effects such as radiated far-fields pattern distortion, reduced bandwidth and polarization mismatch will arise. With the aim of mitigating these degrading effects, solutions have been proposed in the digital domain to compensate for the mutual coupling in compact arrays [3], [4]. However, after the analog to digital conversion, the signal will be distorted by noise, and the degrading effects cannot be compensated for in the digital domain. Alternatively, networks are designed, which are connected to the antenna array, comprised of Radio Frequency (RF) elements, for the decoupling and matching of array elements [5], [6]. This approach ensures the compensation of degrading effects brought by the strong mutual coupling in compact arrays.

For applications that require digital processing of signals, as a subsequent stage of the antenna array, it is important to take into account all the characteristics of the impinging waves such as frequency, bandwidth, polarization, among others. With respect to polarization, we highlight the following: first, it is of crucial importance that the array is dual-polarized in order to mitigate errors due to polarization mismatch with respect to the acquired signal. Depending on the level of mismatch, considerable error may occur which can lead to a severe degradation of the direction finding performance [7], [8]. Second, the Decoupling and Matching Network (DMN) connected to the antenna array may be optimized with respect to polarization. The aim of this paper is to investigate the performance of compact antenna arrays and the corresponding DMN with respect to the signal polarization.

This paper is organized as follows: In Section II, we show in details the design of the dual-polarized compact L-Quad antenna array comprised of DMN for the decoupling and matching of the radiating elements. Section III presents the data model employed and reviews the CRLB as a performance metric for direction finding. Section IV provides simulation results, and finally, we conclude and summarize the work in Section V.

II. Compact Antenna Array Design

A. L-Quad Antenna Array

In this paper we analyse a compact dual-polarized antenna array comprised of crossed dipoles, hereafter referred to as L-Quad antenna array. Since the antenna is dual-polarized, errors in bearing due to mismatch between the signal polarization and the polarization of the antenna elements can be minimized.

The L-Quad element contains four pairs of folded monopoles acting as dipoles, placed orthogonally to each other for dual linear polarization [9]. The monopoles are folded for wider coverage over elevation. The pair of monopoles is excited at the end points instead of at the center, with 180° relative phase difference and is optimized for the frequency band 1.76 GHz–1.84 GHz. Figure 1 depicts the L-Quad element and the radiation patterns (ϑ and φ components) of each port. Figure 2 depicts the antenna array in the circular arrangement.

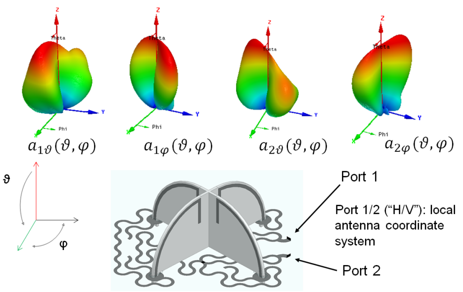

Figure 1: L-Quad circular five-element array.

B. Decoupling and Matching Network

In compact antenna arrays, with inter-element spacing smaller than half of the free-space wavelength, strong electromagnetic coupling gives rise to degrading effects, which degrades the capability of the array for direction finding of emitters, since the DoA estimation is strongly dependent on the radiation characteristics of the antenna array. The mutual coupling gives rise not only to power mismatch, but also to distorted individual radiation patterns [6], [10].

Solutions in the digital domain to compensate for the degrading effects of mutual coupling [3], [4] do not consider the loss of SNR due to power mismatch, since this happens before sampling. As a consequence, performance loss becomes severe for strong coupling. The degraded performance does

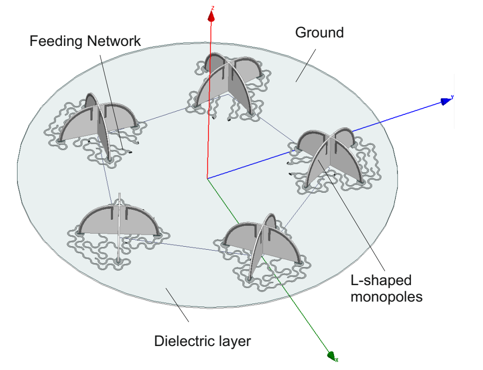

Figure 2: L-Quad antenna array investigated: circular arrangement.

not seem to be a consequence exclusively of mismatch loss at the antenna ports. The array manifold matrix will become more ill-conditioned the higher the coupling between radiating elements. Since the differential information at antenna array outputs is the key as indicator of DoA, it may make sense also to decouple (meaning "compensating the coupling") by lossless circuit elements before sampling, in order to maximize the SNR for this differential information.

By Volmer, [6], decoupling and matching was achieved with the use of distributed elements, which are designed based on the analysis of the array radiation eigenmodes, which complement the S-parameters for the description of the array. It was shown in the above reference that the resulting network of antennas and DMN directly launches the array radiation eigenmodes. This results from the close interaction of the passive DMN with the physical antennas by reactive exchange of energy. So we argue that the DMN together with the physical antennas should be considered as a "new" antenna array. Hence, the output (eigenmodes) can be individually matched and even left unused if their contribution can be considered as minor. Of course, the resulting radiation patterns have to be precisely measured (including all implementation losses) and subsequently used as reference for DoA estimation.

The eigenmode decomposition approach, first introduced by Stein [11] and further detailed in [6], describes the antenna as an array of ideally uncoupled ports and orthogonal radiation patterns. The antenna array radiation pattern is decomposed into fundamental modes of radiation, each with an efficiency that measures the amount of power radiated to the far field, and defined as eigenefficiency. The array eigenefficiencies complement the S-parameters for the analysis of coupling within the array.

Let $\boldsymbol{A}(\vartheta,\varphi) := \begin{bmatrix}\boldsymbol{a}_1(\vartheta,\varphi) & \boldsymbol{a}_2(\vartheta,\varphi)\end{bmatrix}$ be the complex matrix of polarimetric array steering vectors where $\boldsymbol{a}_p(\vartheta,\varphi) \in \mathbb{C}^{M\times 1}$ for $p = 1, 2$, considering a plane wave impinging on an antenna array comprising M elements, l. We start by defining the radiation matrix $\boldsymbol{H} \in \mathbb{C}^{M\times M}$ of the antenna array, when Ohmic losses cannot be neglected within the array.

$$H_{ij} = \frac{1}{4\pi} \oint \boldsymbol{A}_i^{\mathrm{H}}(\vartheta,\varphi)\boldsymbol{A}_j(\vartheta,\varphi) d\Omega \quad (1)$$

where Ω is the solid angle. Since the matrix $\boldsymbol{H}$ is Hermitian, it is possible to diagonalize it through the following transformation. The subscript i, j refer to the antenna array ports.

$$\boldsymbol{\Lambda} = \boldsymbol{Q}^H \boldsymbol{H} \boldsymbol{Q} \quad (2)$$

where $\boldsymbol{\Lambda} \in \mathbb{R}^{M \times M}$ is a diagonal matrix with each diagonal element λ_m representing the m-th mode eigenefficiency and $\boldsymbol{Q} \in \mathbb{C}^{M \times M}$ is the matrix comprised by the M eigenvectors $\boldsymbol{q}_m$, the eigenmodes. The matrix $\boldsymbol{Q}$ characterizes the fundamental modes of radiation of an antenna array and the matrix Λ comprises the correspondent modal radiation efficiencies, the eigenefficiencies.

We observe that the $\boldsymbol{Q}$ guides into the design of the required decoupling network. In order to decouple the antenna array, the goal is to design a decoupling network that renders the excitation given by the eigenvectors. Considering that the new antenna array is comprised of a decoupling network connected to the antenna array ports, the new array manifold for each mode m, denoted by $\tilde{A}_m(\vartheta,\varphi)$ is:

$$\tilde{A}_m(\vartheta,\varphi) = \frac{1}{\sqrt{\lambda_m}} \boldsymbol{q}_m^{\mathrm{T}} \boldsymbol{A}(\vartheta,\varphi) \quad (3)$$

where $\boldsymbol{q}_m, \in \mathbb{C}^{M \times 1}$ is the m-th eigenvector, corresponding to the m-th column of the matrix of eigenmodes $\boldsymbol{Q}$.

The magnitude and phase of the eigenvectors thus define the magnitude and relative phase of excitation at each port, for effective decoupling. The eigenmode excitation may be thus considered as an equivalent to beamforming. Additionally, matching networks may be designed independently for each mode, to ensure optimal power transfer.

III. Direction Finding

A. Data Model

In this paper, the source signals are modeled as stochastic variables with Gaussian zero mean distribution. The polarization of the impinging waves is assumed to be known, and as a consequence, no estimation is carried out with respect to polarization. In a multi-wave scenario, assuming that L snapshots are observed and that N different source waves impinge on the antenna, the measured output of the array is:

$$\boldsymbol{Y} = \sum_{n=1}^{N} \boldsymbol{A}_n \boldsymbol{k}_n \boldsymbol{s}_n^{\mathrm{T}} + \boldsymbol{N} \quad (4)$$

where $\boldsymbol{k}$ is the Jones vector of a linear polarized wave front, $\boldsymbol{k} \in \mathbb{C}^{2 \times 1}$, given by $\boldsymbol{k} := \begin{bmatrix}\cos(\alpha) & \sin(\alpha)\end{bmatrix}^{\mathrm{T}}$.

In (4), $\boldsymbol{N} \in \mathbb{C}^{M \times L}$ is the matrix for the circular symmetric complex Gaussian noise with unknown covariance σ^2, $\boldsymbol{Y} \in \mathbb{C}^{M \times L}$ is the array received matrix and $\boldsymbol{s}_n \in \mathbb{C}^{L \times 1}$ the vector of complex source signals.

B. Polarimetric Cramér-Rao Lower Bound of Decoupled Arrays via Eigenmode Excitation

For the computation of the polarimetric CRLB, we refer to [12]. The expression, for the estimation of DoA, is given by:

$$\mathrm{CRLB}^{-1} = 2T\,\mathrm{SNR}\,\{\tilde{\boldsymbol{D}}^{\mathrm{H}}\tilde{\boldsymbol{D}} - \tilde{\boldsymbol{D}}^{\mathrm{H}}\tilde{\boldsymbol{A}}(\tilde{\boldsymbol{A}}^{\mathrm{H}}\tilde{\boldsymbol{A}})^{-1}(\tilde{\boldsymbol{D}}^{\mathrm{H}}\tilde{\boldsymbol{A}})^{\mathrm{H}}\} \quad (5)$$

where $\tilde{\boldsymbol{D}}$ comprises the derivatives of the array manifold of the decoupled array, with respect to the directions ϑ and φ.

In this section, we present details on the computation of the CRLB to the DoA estimation of impinging waves using an array decoupled by eigenmode excitation. Let us begin by analyzing how the derivatives of the modal eigen-patterns with respect to DoA behave. To this end, we may write the following expression for the derivative of the decoupled array manifold with respect to ϑ (analogous procedure for the derivative with respect to φ), recalling (3).

$$\frac{\partial}{\partial\vartheta}\tilde{A}_m(\vartheta,\varphi) = \frac{1}{\sqrt{\lambda_m}} \boldsymbol{q}_m^{\mathrm{T}} \frac{\partial}{\partial\vartheta} \boldsymbol{A}(\vartheta,\varphi) \quad (6)$$

It is easy to observe that the derivatives of the orthogonal modes present a beamforming with weights given by the eigenmodes. The matrix of the derivatives of each modal radiation pattern may be finally written with respect to the derivatives matrix of the antenna array manifold $\boldsymbol{D}$ as:

$$\tilde{\boldsymbol{D}} = \boldsymbol{\Lambda}^{-1/2} \boldsymbol{Q}^{\mathrm{T}} \boldsymbol{D} \quad (7)$$

where $\boldsymbol{\Lambda}^{-1/2}$ is a diagonal matrix comprised by the square root of each λ_m.

From (5), recalling the fact that $\boldsymbol{Q}$ is unitary, i.e. $\boldsymbol{Q}^{\mathrm{H}}\boldsymbol{Q} = 1$, and that the radiation matrix $\boldsymbol{H}$ is Hermitian, we derive the CRLB for a decoupled array using the eigenmode approach:

$$\begin{aligned}\tilde{\mathrm{CRLB}}^{-1} &= 2L\,\mathrm{SNR}\,\{\boldsymbol{D}^{\mathrm{H}}(\boldsymbol{H}^{\mathrm{T}})^{-1}\boldsymbol{D} - \boldsymbol{D}^{\mathrm{H}}(\boldsymbol{H}^{\mathrm{T}})^{-1} \\ &\times \boldsymbol{A}(\boldsymbol{A}^{\mathrm{H}}(\boldsymbol{H}^{\mathrm{T}})^{-1}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{H}}(\boldsymbol{H}^{\mathrm{T}})^{-1}\boldsymbol{D}\}\end{aligned} \quad (8)$$

IV. Simulation Results

In Section II, we discussed the procedures for the design of a decoupling network comprised of distributed elements using eigenmode approach. Equation (1) computes the antenna array radiation matrix $\boldsymbol{H}$, which is to be decomposed into eigenvectors and corresponding eigenefficiencies, guiding the design of the decoupling network.

Note that in (1), the elements of the radiation matrix $\boldsymbol{H}$ are computed as an integral over all possible elevation and azimuth angles. Since the decoupling network is designed from the eigenvectors resulting from the decomposition of $\boldsymbol{H}$, it follows that it is optimized for all possible azimuth and elevation angles. Moreover, note that in (1), the product $\boldsymbol{A}_i^{\mathrm{H}}(\vartheta,\varphi)\boldsymbol{A}_j(\vartheta,\varphi)$ considers both polarization components with same weight for the computation of each element H_{ij} in the radiation matrix. However, if it is known a priori that waves are only impinging on the array e.g. with vertical polarization, the decoupling network could be optimized by assuming only the vertical component of the radiation patterns in (1).

Figures 3 – 5 show results for the CRLB when one wave impinges on the five-element dual-polarized L-Quad array with three possible polarization states and the decoupling network is designed taking into account the signal's Jones vector. The figures compare the results obtained when the design of the decoupling network consider the signal polarization state to the results obtained when the signal polarization is not taken into account for the computation of $\boldsymbol{H}$.

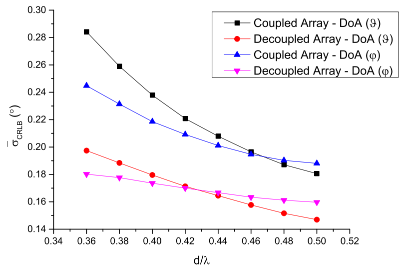

Figure 3: CRLB for one impinging wave and Jones vector defined by $\beta = 45°$ and $\phi = 0°$ – linear polarization.

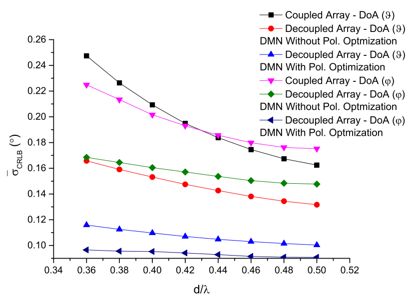

Figure 4: CRLB for one impinging wave and Jones vector defined by $\beta = 0°$ and $\phi = 0°$, vertical polarization.

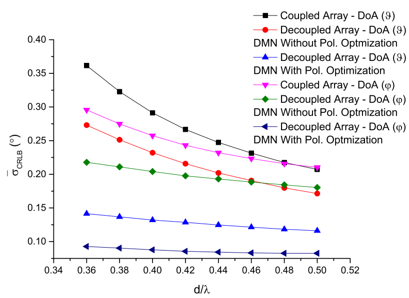

Figure 5: CRLB for one impinging wave and Jones vector defined by $\beta = 90°$ and $\phi = 0°$, horizontal polarization.

It is clear to observe that the performance of the decoupling network is optimized for direction finding if the signal polarization is accounted for when computing the array radiation matrix. However, situations when the impinging wave's polarization is known a priori are unrealistic. Moreover, when multiple waves are being received by the array, each with a different polarization, it is clear that an optimal design must consider all possible polarization states, and hence, the computation of $\boldsymbol{H}$ must be carried out as in (1).

V. Conclusion

In this work, we investigated the performance in terms of direction finding of a compact L-Quad full-polarimetric antenna array connected to a DMN for the mitigation of degrading effects that arise from strong electromagnetic interference between closely spaced radiating elements. The capability of the array for direction finding of emitters was evaluated with respect to the CRLB.

If the scenario allows us to know the signal polarization a piori, and if we use this information for the computation of the radiation matrix, implying that we only consider the contribution of the array response in the polarization that matches the signal, we observe that the performance of the decoupling network is optimized for direction finding, giving rise to lower CRLB. However, in situations such as when multiple waves are being received by the array, each with a different polarization, it is clear that an optimal design must consider the full array radiation response.

References

[1] D. Schulz and R. Thomä, "Search-based music techniques for 2d doa estimation using eadf and real antenna arrays," in *ITG Workshop on Smart Antennas*, march 2013.

[2] M. Pralon, D. Schulz, and R. S. Thomä, "An eigen-analysis of a compact l-quad antenna array for direction finding," in *Antennas and Propagation in Wireless Communications (APWC), 2014 IEEE-APS Topical Conference on*, 2014, pp. 369–372.

[3] Q. Bao, C. C. Ko, and W. Zhi, "DoA Estimation Under Unknown Mutual Coupling and Multipath," *IEEE* Transactions on Aerospace and Electronic Systems, vol. 41, no. 2, pp. 565–573, Apr. 2005.

[4] A. J. Weiss, and B. Friedlander, "Direction Finding in the Presence of Mutual Coupling," *IEEE* Transactions on Antennas and Propagation, vol. 39, no. 2, pp. 273–284, 1991.

[5] J. Weber, C. Volmer, K. Blau, R. Stephan, and M. A. Hein, "Miniaturized antenna arrays using decoupling networks with realistic elements," *IEEE* Transactions on Microwave Theory and Techniques, vol. 54, no. 6, pp. 2733-–2740, June, 2006.

[6] C. Volmer, J. Weber, R. Stephan, K. Blau, and M. A. Hein, "An Eigen-Analysis of Compact Antenna Arrays and Its Application to Port Decoupling," *IEEE* Transactions on Antennas and Propagation, vol. 56, no. 2, pp. 360—370, February, 2008.

[7] M. Landmann, M. Käske, and R. S. Thomä, "Impact of Incomplete and Inaccurate Data Models on High Resolution Parameter Estimation in Multidimensional Channel Sounding," *IEEE* Transactions on Antennas and Propagation, vol. 60, no. 2, pp. 557-–573, 2012.

[8] M. Hage, and M. Oispuu, "The effect of imperfect dual-polarized array elements on DOA and polarization estimation," in *International Radar Symposium*, 2011.

[9] Q. Chen, and R. Liu, "C. Domíngez, F. Tiezzi, R. Wansch, R. Rummel, and A. Popugaev, "Low-cost s-band antennas for mobile satellites systems, in *5th European Conference on Antennas and Propagation*, 2011.

[10] S. Irteza, N. Murtaza, S. Caizzone, R. Stephan, and M. A. Hein, "Compact planar L-band antenna arrays with optimal diversity performance," in Proc. IEEE-APS Topical Conference on Antennas and Propagation in Wireless Communications (APWC), pp. 512–515, 2011.

[11] S. Stein, "On cross coupling in multiple-beam antennas," in *IRE* Trans. Antennas Propagation, September, 1962, vol. 10, no. 5, pp. 548–557.

[12] D. Schulz and R. Thomä, "Cramér-Rao Lower Bounds for Polarimetric 2D Direction of Arrival Estimation," in *ITG Workshop on Smart Antennas*, March 2015, pp. 1–8, Ilmenau, Germany.