Dispersion of THz Modes Localized on Layered Superconductor Controlled by DC Magnetic Field

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***Abstract*— Being materials that support terahertz waves propagation, layered superconductors attract great attention of many researchers. The Josephson plasma modes localized on a slab of layered superconductor can possess an anomalous dispersion. The possibility of the anomalous dispersion manipulation opens wide perspectives for applications. One of the tools that can flexibly change the electromagnetic properties of layered superconductors is the external DC magnetic field. The effect of the DC magnetic field on the spectrum of the localized modes is a subject of the present paper. We present the derivation and analysis of the dispersion relations and discuss some interesting corollaries such as resonant amplification of wave transmission induced by the internal excitation of the localized modes and the possibility of the internal reflection of the localized modes controlled by the external DC magnetic field.**

***Keywords*— high-temperature superconductors, Josephson junctions, electromagnetic fields, Terahertz metamaterials.**

I. Introduction

Terahertz (THz) frequency technologies receive great attention of many researchers for more than a decade due to promising applications in physics, chemistry, astronomy, security systems, medical diagnostics, and environmental control (see, e.g., [1]). One of the ways to obtain electromagnetic radiation in THz frequency range is to use Josephson junctions. The emission from one junction is weak, however a stack of the Josephson junctions can be used in generation of a strong coherent THz pulse. The latter was realized experimentally (see, e.g., [2], [3]) using layered superconductors.

As was proven by experimental studies [4], [5], layered superconductors can be considered as periodic materials, in which the thin superconducting layers are separated by the thicker insulator ones and are electrodynamically related to each other by means of the intrinsic Josephson effect. High-temperature superconductors based on Bi, La or Y with CuO_2 superconducting layers are examples of such materials. The current-carrying capability of layered superconductors is strongly anisotropic. The current along the layers is of the same nature as in the bulk superconductors, while the cross current is caused by the Josephson effect. This anisotropic solid state plasma supports propagation of the specific excitations in THz frequency range — the Josephson plasma waves (JPWs).

It worth noticing that layered superconductors can be used not only for the generation of THz emission. Studying the interaction of intense THz pulses with layered superconductors can be used for JPWs manipulation and even open new possibilities for high-temperature superconductive state control (see experimental works [6], [7]).

In article [8], it was shown theoretically that the anomalous dispersion of JPWs localized on the slab of layered superconductor can be observed in a certain range of frequencies and wave numbers. The ability of the anomalous dispersion manipulation can find various applications. A flexible tool that can tune the electromagnetic properties of layered superconductor, and by that, modes propagation on it, is the external DC magnetic field. The effect of the DC magnetic field on the THz waves transmission and reflection from layered superconductors was studied recently in [9], [10]. In this work, we study theoretically the effect of the relatively weak external DC magnetic field on the spectral properties of the localized JPWs.

II. Theoretical model

We study linear localized JPWs propagating along a slab of layered superconductor sandwiched between two dielectric half-spaces (see Fig. 1). The layers are perpendicular to the interfaces of the slab. The external DC magnetic field of the magnitude H_0 is directed along the layers and the interfaces of the slab.

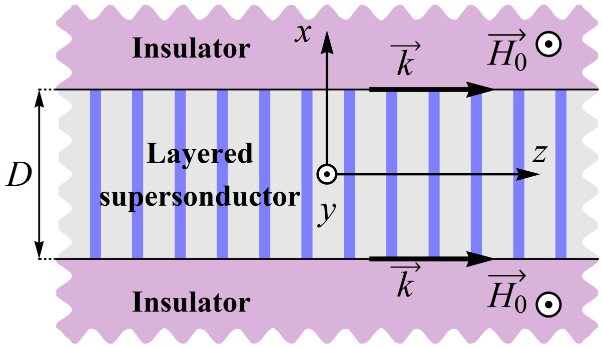

Fig. 1. Sketch of the setup, where D is the thickness of the sample, $\vec{H}_0$ is the external DC magnetic field, $\vec{k}$ is the wave vector.

We consider the localized JPWs of the following polarization:

$$\vec{H}(x,y,z,t) = \{0, H_y(x), 0\} \exp(ik_z z - i\omega t),$$
$$\vec{E}(x,y,z,t) = \{E_x(x), 0, E_z(x)\} \exp(ik_z z - i\omega t), \quad (1)$$

where ω is the frequency of the localized mode that propagates along the z-axis, i.e. $k_y = 0$.

The electromagnetic field in the dielectrics can be found from Maxwell equations, considering that the waves evanesce far from the slab. The electromagnetic field in the layered superconductor can be described by the coupled sine-Gordon equations for the gauge-invariant interlayer phase difference φ between neighboring layers [11]. Assuming the period d of the layered superconductor to be much smaller than the wavelength across the layers, $k_z d \ll 1$, these coupled equations can be presented in the continual limit as,

$$\left(1 - \lambda_{ab}^2 \frac{\partial^2}{\partial z^2}\right)\left[\frac{1}{\omega_J^2}\frac{\partial^2\varphi}{\partial t^2} + \sin\varphi\right] - \lambda_c^2 \frac{\partial^2 \varphi}{\partial x^2} = 0, \quad (2)$$

where $\omega_J = (8\pi e d J_c/\hbar\varepsilon_s)^{1/2}$ is the Josephson plasma frequency, λ_{ab} and $\lambda_c = c/\omega_J \varepsilon_s^{1/2}$ are the London penetration depths across and along the layers, respectively, J_c is the maximal value of the Josephson current density, and ε_s is the dielectric constant of the insulating layers in the sample.

Subsequently, the z-component of electric E_z and y-component of magnetic H_y fields are related to the phase difference φ by the following relations,

$$\frac{\partial H_y}{\partial x} = -\frac{\mathcal{H}_0}{\lambda_c}\left[\sin\varphi + \frac{1}{\omega_J^2}\frac{\partial^2 \varphi}{\partial t^2}\right], \quad E_z = \frac{\mathcal{H}_0}{\omega_J \sqrt{\varepsilon_s}}\frac{\partial \varphi}{\partial t}, \quad (3)$$

where $\mathcal{H}_0 = \Phi_0/2\pi d\lambda_c$, and $\Phi_0 = \pi c\hbar/e$ is the magnetic flux quantum.

A. Localized JPW in the Layered Superconductor

The external DC magnetic field penetrates inside the sample from two interfaces and affects the distribution of the gauge-invariant interlayer phase difference φ. We represent φ as a sum of static solution $\varphi_\pm$ caused by the DC magnetic field and a small additive φ_{lm} caused by the localized mode,

$$\varphi(x,z,t) = \varphi_\pm(x) + \varphi_{lm}(x,z,t), \quad (4)$$

Here we study relatively small magnetic fields, $H_0 < \mathcal{H}_0$, when the Josephson vortices do not wholly penetrate inside the sample. In this case, the DC magnetic field penetrates inside the sample in form of fictitious soliton tails from both interfaces. We assume the sample to be sufficiently thick, $\exp(D/\lambda_c) \gg 1$, so that the DC magnetic fields penetrating from the two interfaces do not interact. Solving sine-Gordon equation (2) in this static case, one can obtain well-known (see e.g. [12]) soliton solution,

$$\varphi_\pm(\xi) = \mp 4 \arctan\left[\exp\left(\xi_0 \pm \xi\right)\right], \quad (5)$$

where superscripts $+$ and $-$ mean the upper and lower interfaces, respectively, near which the soliton tails exist, and $\xi = x/\lambda_c$ is normalized coordinate. The constant $\xi_0 = \delta + \operatorname{arcosh}(h_0^{-1})$ corresponds to the center of the fictitious vortex and is defined by the normalized magnitude $h_0 = H_0/\mathcal{H}_0$ of the external DC magnetic field and the normalized half-thickness $\delta = D/2\lambda_c$ of the slab.

For the localized mode $\varphi_{\rm lm}(\xi,z,t)$, we seek the solution in the form of the wave propagating along the z-axis,

$$\varphi_{\rm lm}(\xi,z,t) = a(\xi)\exp[ik_z z - i\omega t)]. \quad (6)$$

Substituting (6) into sine-Gordon equation (2), one can obtain differential equation for amplitude $a(\xi)$,

$$\frac{1}{1+\kappa_z^2} a''(\xi) + \left[\Omega^2 - 1 + u_+(\xi) + u_-(\xi)\right] a(\xi) = 0,$$

where $u_\pm(\xi) = 2/\cosh^2(\xi_0 \pm \xi)$, the prime denotes derivative with respect to ξ, $\Omega = \omega/\omega_J$ is normalized frequency, $\kappa_z = k_z \lambda_{ab}$ is the normalized z-projection of the wave vector.

Using (3), we can express the components H_y^s and E_z^s of the electromagnetic field in the slab via the function $a(\xi)$,

$$H_y^s(\xi) = \frac{\mathcal{H}_0 a'(\xi)}{1+\kappa_z^2}, \quad E_z^s(\xi) = -\frac{i\Omega\mathcal{H}_0 a(\xi)}{\sqrt{\varepsilon}}. \quad (7)$$

B. Dispersion Relation

The symmetry of the studied system implies the symmetry of the localized modes, symmetric and antisymmetric with respect to the magnetic field. The symmetric and antisymmetric solutions of (7) can be expressed in terms of the associated Legendre functions,

$$a(\xi) = a_{\rm sym}\left[\frac{f_p(\xi_0-\xi)}{f_p(\xi_0)} - \frac{f_q(\xi_0-\xi)}{f_q(\xi_0)}\right], \quad (8)$$
$$a(\xi) = a_{\rm asym}\left[\frac{f_p(\xi_0-\xi)}{f'_p(\xi_0)} - \frac{f_q(\xi_0-\xi)}{f'_q(\xi_0)}\right],$$

respectively. Here $a_{\rm sym}$ and $a_{\rm asym}$ are integration constants, and

$$f_p(\xi) = P_\nu^\mu[\tanh(\xi)], \quad f_q(\xi) = Q_\nu^\mu[\tanh(\xi)], \quad (9)$$

where $P_\nu^\mu[\tau]$ and $Q_\nu^\mu[\tau]$ are associated Legendre functions of the first and second kind, respectively, with

$$2\nu + 1 = \sqrt{8(\Omega^2-1)^{-1}\kappa_s^2 + 1}, \quad \mu = i\kappa_s, \quad (10)$$

where parameter κ_s represents the normalized x-projection of the wave vector in the absence of the DC magnetic field, $\kappa_s^2 = (\Omega^2 - 1)(1+\kappa_z^2)$.

To derive the dispersion relation for the localized JPWs, one can match the tangential components of the electromagnetic fields at the interfaces of the slab. Therefore, the dispersion relation takes the following form,

$$\frac{a'(\xi=\delta)}{a(\xi=\delta)} = \frac{\varepsilon^{-1}\Omega^2\kappa_s^2}{(\Omega^2-1)\kappa_d}, \quad (11)$$

where κ_d is normalized spatial decrement for the dielectric half-spaces $\kappa_d^2 = \gamma^2\kappa_z^2 - \varepsilon^{-1}\Omega^2 > 0$, $\gamma = \lambda_c/\lambda_{ab}$ is the anisotropy parameter, and $\varepsilon = \varepsilon_s/\varepsilon_d$.

III. Results Discussion

Figures 2 and 3 show the dispersion curves defined by Eqs. (8) and (11) for low frequencies, $\Omega < 1$, and high frequencies, $\Omega > 1$, respectively. The curves at $n = 1$ and $n = 2$ are very close to each other. So to avoid misunderstanding, at the low frequencies we show curves for $n = 1$ only. All the curves gradually move to low frequencies with the increase of the external DC magnetic field. The filled region between the curves shows areas where the propagation of JPWs is possible at given parameters, while the normalized DC magnetic field h_0 is depicted by the color grade. One can see that the dispersion curves are non-monotonous and consist of the parts with normal (where $\partial\Omega/\partial\kappa_z > 0$) and anomalous (where $\partial\Omega/\partial\kappa_z < 0$) dispersions. Therefore, the curves have maximums, where the group velocity vanishes, $\partial\Omega/\partial\kappa_z = 0$. These maximums appear near the light line, $\Omega = \varepsilon^{1/2}\gamma\kappa_z$, and shift when changing h_0.

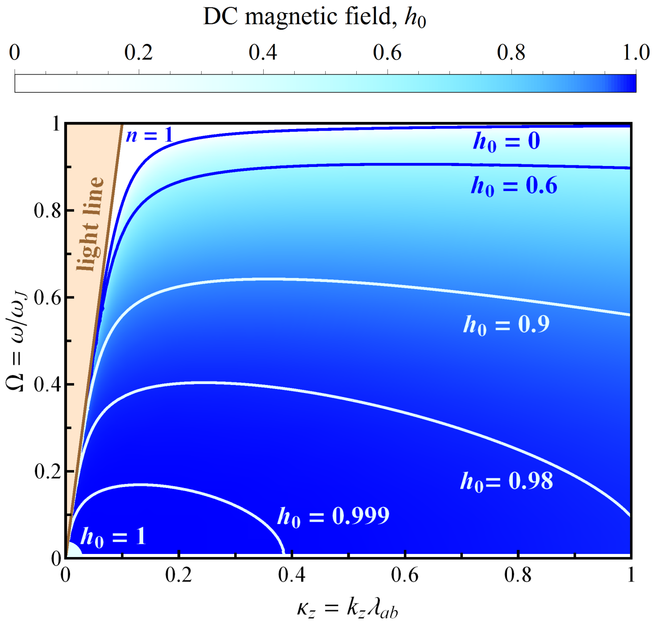

Fig. 2. The dispersion curves with $n = 1$ (antisymmetric localized mode) for $\Omega < 1$ and the normalized DC magnetic field $h_0 = 0$, $h_0 = 0.6$, $h_0 = 0.9$, $h_0 = 0.98$, $h_0 = 0.999$, and $h_0 = 1$, where $h_0 = H_0/\mathcal{H}_0$. The color of the filled area corresponds to the required value of h_0. The filling above the light line, $\Omega = \varepsilon^{1/2}\gamma\kappa_z$, denotes the forbidden zone, where the localized modes do not exists. Other parameters: $\gamma = 5$, $\delta = 5$, $\varepsilon = 4$.

One can employ the external DC magnetic field for the localized JPWs manipulation. The interesting corollaries of the effect are discussed in the following subsections.

A. *Resonant Transmission Amplification*

Let us consider a system that consists of the layered superconductor slab separated from two dielectric leads of permittivity ε_L by the gaps filled with dielectric of permittivity ε_d (see Fig. 4). Assuming the leads to be optically denser than the gaps, i.e. $\varepsilon_L > \varepsilon_d$, we study the transmission of the THz wave through the system. If the incident angle exceeds the angle $\theta_{\text{tot}} = \arcsin(\sqrt{\varepsilon_d/\varepsilon_L})$ of the total internal

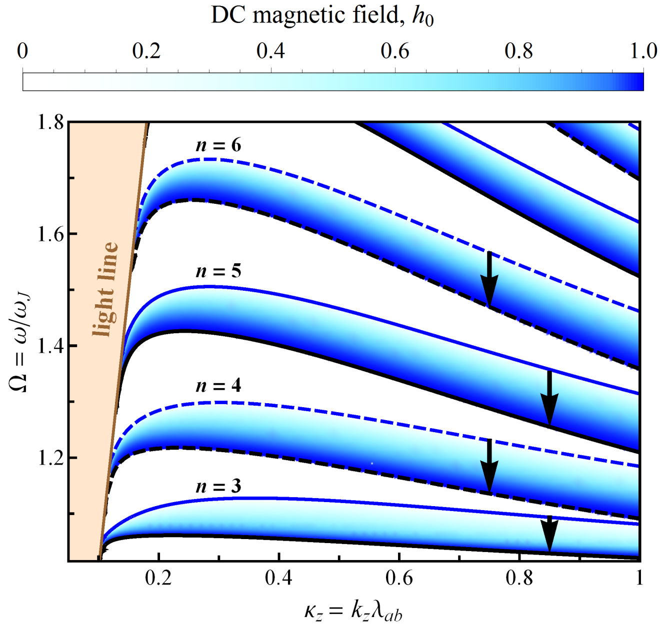

Fig. 3. The dispersion curves for $\Omega > 1$ and the DC magnetic field $h_0 = 0$ and $h_0 = 1$ plotted by solid ($n = 3, 5, \ldots$, antisymmetric localized modes) and dashed ($n = 4, 6, \ldots$, symmetric localized modes) lines. The arrows show the increase of the field h_0. The other parameters and notations are the same as in Fig. 2.

reflection at the "lead–gap" interface, the wave could not propagate in the gap and evanesces inside it. Regularly, this implies that transmission is exponentially suppressed, which can be seen from the dashed curve in Fig. 4. However, when the frequency Ω and z-component of wave vector κ_z of the incident wave nearly satisfy the dispersion relation, the localized JPW can be excited, see the solid line in Fig. 4. In this case, the resonant amplification of transmission should be observed.

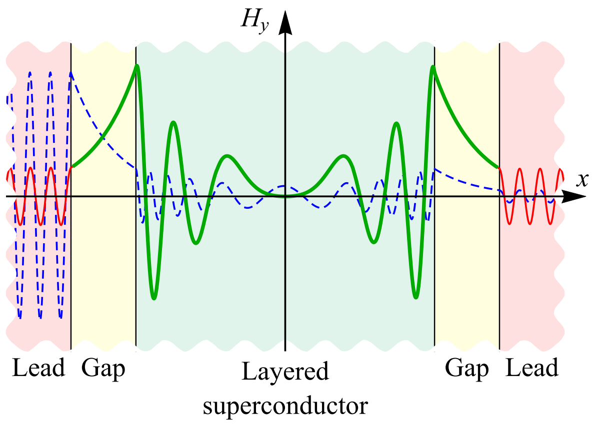

Fig. 4. Principal distribution of the magnetic field in the wave transferred through the system. The regular and resonant transmission is shown by the dashed and solid lines, respectively.

This well-known resonant phenomenon displays some specific features related to the anomalous dispersion of JPWs (see [13]). In addition, the nonlinearity of the Josephson plasma in the layered superconductors enables the JPWs control by means of the external DC magnetic field. Namely, one can achieve the resonance by adjusting the value of the DC field applied to the system. The color-graded regions in Figs. 2 and 3 correspond to the areas where such adjusting is possible. An example of mentioned resonant transmission is shown by solid curves in Fig. 4. It is worth noticing that Fig. 4 is plotted for finite h_0. Since the DC magnetic field penetrates into the layered superconductor in the form of the tails of the fictitious vortices, it causes the change of the amplitude and wavelength of the JPWs near the interfaces.

The suggested resonant phenomenon opens wide prospects for possible applications, such as THz waves filtering, or tuning the emission and receiver frequency from THz sources.

B. Internal reflection of the localized modes

The discussed dispersion curves of localized JPWs are non-monotonous as functions $\Omega(\kappa_z)$ for fixed value of h_0. So, for fixed value of Ω, there can be two values of κ_z on each dispersion curve which correspond to the parts with the normal and anomalous dispersions. This means that the dispersion curves presented as functions $\kappa_z(h_0)$ for fixed value of Ω should be two-valued. This feature is shown in Fig. 5.

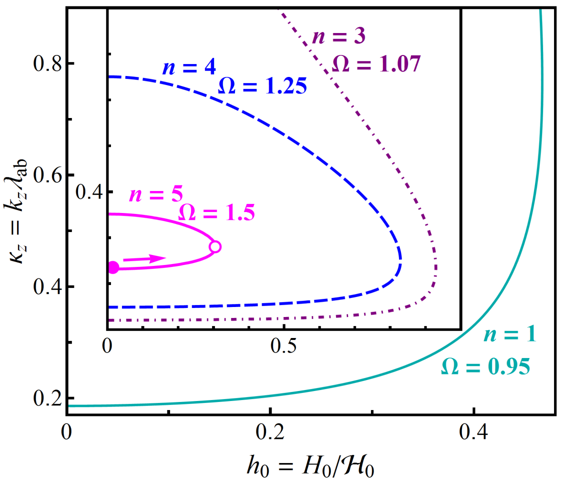

Fig. 5. The dispersion curves $\kappa_z(h_0)$ at the fixed frequencies Ω. Main panel: the curve with number $n = 1$ (solid line) at $\Omega = 0.95$. Inset: the curves with numbers $n = 5$ (solid line), $n = 4$ (dashed line), and $n = 3$ (dash-dotted line) at $\Omega = 1.5$, $\Omega = 1.25$ and $\Omega = 1.07$, respectively. Other parameters are the same as in Fig. 2.

Now considering that the localized mode propagates along the slab of the layered superconductor and that the external DC magnetic field is non-homogeneous and smoothly increases along the z-axis from $h_0 = 0$, we examine the solid curve in inset of Fig. 5. When the external DC magnetic field increases along the propagation, κ_z traces the curve (along the arrow in Fig. 5) from the initial point (marked as a solid circle) to the critical point (marked as an empty circle) with $h_0 = h_{\max} = 0.31$ and $\kappa_z = 0.28$. After the critical point, where h_0 continues to increase, the value of wave vector becomes complex, so the wave attenuates along the z-axis. The study of the structure of complex modes formed in this case goes beyond the scope of the current work. In the critical point the localized mode should reflect, i.e. the phenomenon similar to the internal reflection occurs. This effect can be used to manipulate the localized THz modes propagation.

IV. Conclusion

In the present work, the effect of the external static magnetic field on the spectral properties of THz modes localized on layered superconductors has been studied theoretically. We have derived analytically the dispersion relation and presented the numerical analysis of the results. We have shown that the dispersion is anomalous in a wide range of the parameters and can be flexibly tuned by the DC magnetic field. Also, we discuss some possible applications related to the described phenomenon, such as resonant amplification of the transmission and internal reflection of the plasma mode. They can be used in THz electronics to tune emission or receiver frequency and to control the localized mode propagation using the external DC magnetic field.

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